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A closed-form solution for the thunderstorm gust response factor

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SUMMARY:
The Davenport gust factor technique is an efficient approach for the estimate of the wind loading on structures due to synoptic winds and the availability of closed-form solutions favoured its implementation in codes and standards. Starting from an evolutionary power spectral density model consistent with a large database of full-scale wind velocity records, the gust response factor was generalized for thunderstorm outflows based on the definition of suitable equivalent parameters, able to account for the nonstationary characteristics of the wind speed. The present paper outlines the derivation of a closed-form solution for the equivalent parameters, starting from a suitable approximation of the time-varying variance of the response. The gust response factor derived through the closed-form solution proposed is compared with numerical solution and with the one existing formulation from literature. Results show that the proposed solution is very accurate, providing an efficient and handy tool for the wind-resistant structural design.

Keywords: Thunderstorm outflows, Gust response factor, Closed-form solution

1. INTRODUCTION

The gust factor technique developed by Davenport (1967) constitutes nowadays a well-established approach for the estimate of the wind actions and wind-excited response of structures under synoptic winds. The introduction of closed-form solutions (e.g. Solari, 1993) favoured its usage for rapid engineering calculations and code applications (Solari and Kareem, 1998).
In the literature, the generalization of the gust factor technique to thunderstorm outflows was investigated following different approaches. Introducing the Gust Front (GF) factor, Kwon and Kareem (2019) proposed a closed-form solution that allows to derive the gust response factor under the hypothesis of long pulse duration, hence neglecting the transient dynamic effects due to the nonstationary turbulence, and in condition of absence of background wind.
However, Roncallo et al. (2022) and successively Roncallo and Tubino (2023) showed that such hypothesis can be overconservative and rather unrealistic, especially for lowly-damped systems.
With the aim of overcoming these limitations, this paper outlines the derivation of a closed-form solution for the thunderstorm gust response factor, able to account for the transient dynamic effects and the role of the two parameters defining the slowly-varying mean wind velocity, i.e. the background wind and the duration of the intense phase of the thunderstorm outflow.

2. ANALYTICAL FRAMEWORK

Let us consider a linear elastic SDOF system that can be schematized as a point-like surface with area A perpendicular to the wind velocity and drag coefficient c_D, characterized by mass m, fundamental circular frequency $\omega_0 = 2\pi n_0$ (being n_0 the natural frequency and $T_0 = 1/n_0$ the natural period) and damping ratio ξ. Assuming it is subjected to the wind action provided by a thunderstorm outflow modelled as proposed by Roncallo et al. (2022), the maximum value of the alongwind displacement $x(t)$ is given by:

$$x_{\max} = G_x \bar{x}_{\max} \tag{1}$$

where $\bar{x}_{\max} = \rho \bar{v}_{\max}^2 A c_D \big/ 2m(2\pi n_0)^2$ is the maximum value of the mean part of the response, with ρ the air density and $\bar{v}_{\max}$ the maximum value of the slowly-varying mean wind velocity, and G_x is the thunderstorm gust response factor (Roncallo and Tubino, 2023):

$$G_x = 1 + 2 I_v g_x \left(\tilde{\nu}_x \tilde{T}_{eq} \right) \sqrt{B^2 + R^2} C \tag{2}$$

where I_v the mean value of the turbulence intensity, g_x the Davenport peak factor, $\tilde{\nu}_x = \nu_x / n_0$ the normalized expected frequency of the response, $\tilde{T}_{eq}$ the non-dimensional equivalent period, C the non-dimensional equivalent standard deviation (Roncallo et al., 2022; Roncallo and Tubino, 2023), B and R the background and resonance factor, respectively, available in closed-form (Davenport, 1967; Solari, 1993). A closed-form solution for the generalized gust response factor in Eq. (2) requires the estimate of the parameters C and $\tilde{T}_{eq}$ (Michaelov et al., 2001) which are defined as follows (Roncallo and Tubino, 2023):

$$C^2 = \int_{-\tilde{T}_{\max}/2}^{+\tilde{T}_{\max}/2} c_{00,\tilde{x}'}^5 \left(\tilde{t} \right) d\tilde{t} \bigg/ 4 I_v^2 \left(B^2 + R^2 \right) \int_{-\tilde{T}_{\max}/2}^{+\tilde{T}_{\max}/2} c_{00,\tilde{x}'}^4 \left(\tilde{t} \right) d\tilde{t} \tag{3}$$

$$\tilde{T}_{eq} = \int_{-\tilde{T}_{\max}/2}^{+\tilde{T}_{\max}/2} c_{00,\tilde{x}'}^4 (\tilde{t}) d\tilde{t} \bigg/ [4 I_v^2 \left(B^2 + R^2 \right) C^2]^4 \tag{4}$$

where $\tilde{t} = t / T_0$ is a non-dimensional time, $\tilde{T}_{\max} = T_{\max} / T_0$ with $T_{\max} = 600s$, and $c_{00,\tilde{x}'}(\tilde{t})$ the first non-dimensional Non-Geometrical Spectral Moment (NGSM) of the response:

$$c_{00,\tilde{x}'}(\tilde{t}) = \int_0^{+\infty} \left| Z\left(\tilde{n}, \tilde{t} \right) \right|^2 S_{\tilde{v}'}(\tilde{n}) d\tilde{n} \tag{5}$$

with $S_{\tilde{v}'}(\tilde{n})$ the dimensionless power spectral density of the reduced turbulence modelled through the spectral model by Solari and Piccardo (2001) and $Z(\tilde{n}, \tilde{t})$ the dimensionless evolutionary frequency response function (Roncallo et al., 2022). Since an analytical solution of Eqs. (3)-(4) is computationally demanding, an approximated solution is searched.

3. CLOSED-FORM SOLUTION

A closed-form solution for the equivalent parameters C and T_{eq} (Eqs. (4) and (5)) is obtained by approximating the NGSM as follows:

$$c_{00,\tilde{x}'}(\tilde{t}) = 2I_v \hat{\gamma}^4(\tilde{t})\sqrt{B^2+R^2} \quad (6)$$

where the function $\hat{\gamma}^4(\tilde{t})$ represents the modulation in time of the first NGSM, accounting for the transient dynamic effects. The following approximated expression is assumed:

$$\hat{\gamma}^4(\tilde{t}) = \begin{cases} \gamma^{*4}, \ \tilde{t} < \tilde{T}_1, \ \tilde{t} > \tilde{T}_2 \\ \Lambda + \Phi\tilde{t}/\tilde{T}_1, \ -\tilde{T}_1 \le \tilde{t} < 0, \\ \Lambda - \Phi\tilde{t}/\tilde{T}_2, \ 0 \le \tilde{t} \le \tilde{T}_N \end{cases} \qquad \tilde{T}_N = \begin{cases} \tilde{T}_2, \ \tilde{T}_2 \le \tilde{T}_{\max}/2 \\ \tilde{T}_{\max}/2, \ \tilde{T}_2 > \tilde{T}_{\max}/2 \end{cases} \quad (7)$$

where γ^* is a measure of the intensity of the background wind (Roncallo and Solari, 2020), $\tilde{T}_1 = \tilde{T}/2$, $\tilde{T}_2 = \tilde{T}_1[1+1/(2\xi\tilde{T})]$, $\Lambda = \Phi + \gamma^{*4}$ and $\Phi = \beta(1-\gamma^{*4})$, being $\tilde{T} = T/T_0$ (with T the duration of the intense phase of the outflow) and $\beta = 1/[1+1/(4\xi\tilde{T})]$. Substituting Eqs. (6)-(7) into Eqs. (3)-(4) it follows:

$$C^2 = \frac{\gamma^{*20}\left(\frac{\tilde{T}_{\max}}{\tilde{T}_{r,0}} - 1\right) + \left(\Lambda^5 - \frac{5}{2}\Lambda^4\Phi + \frac{10}{3}\Lambda^3\Phi^2 - \frac{5}{2}\Lambda^2\Phi^3 + \Lambda\Phi^4 - \frac{1}{6}\Phi^5\right)}{\gamma^{*16}\left(\frac{\tilde{T}_{\max}}{\tilde{T}_{r,0}} - 1\right) + \left(\Lambda^4 - 2\Lambda^3\Phi + 2\Lambda^2\Phi^2 - \Lambda\Phi^3 + \frac{1}{5}\Phi^4\right)} \quad (8)$$

$$\tilde{T}_{eq} = \left[\gamma^{*16}\left(\tilde{T}_{\max} - \tilde{T}_{r,0}\right) + \tilde{T}_{r,0}\left(\Lambda^4 - 2\Lambda^3\Phi + 2\Lambda^2\Phi^2 - \Lambda\Phi^3 + \frac{1}{5}\Phi^4\right)\right] \Big/ \left[C^2\right]^4 \quad (9)$$

with $\tilde{T}_{r,0} = \tilde{T}_1 + \tilde{T}_N$. The gust response factor can then be obtained in closed-form by substituting Eqs. (9) and (10) into Eq. (3). It should be mentioned that, differently from the literature (Kwon and Kareem, 2019), the proposed closed-form solution is able to account for a wide range of different thunderstorm cases by choosing the parameters γ^* and T while accounting the transient dynamics with a simple mathematical formulation.

4. COMPARISON WITH NUMERICAL SOLUTIONS AND EXISTING FORMULATION

Fig. 1 compares the proposed formulation with the numerically estimated gust response factor and with the only closed-form solution available in literature, to the author's best knowledge, by Kwon and Kareem (2019). In order to provide a proper comparison, the value $\gamma^* = 0$ is fixed as assumed by the Kwon and Kareem (2019) model while $T = 169.81\text{s}$ (Roncallo and Tubino, 2023). From Fig. 1 it can be observed that for high values of ξ and n_0 both solutions are reliable, while for lower values of these parameters the proposed closed-form solution is more accurate.

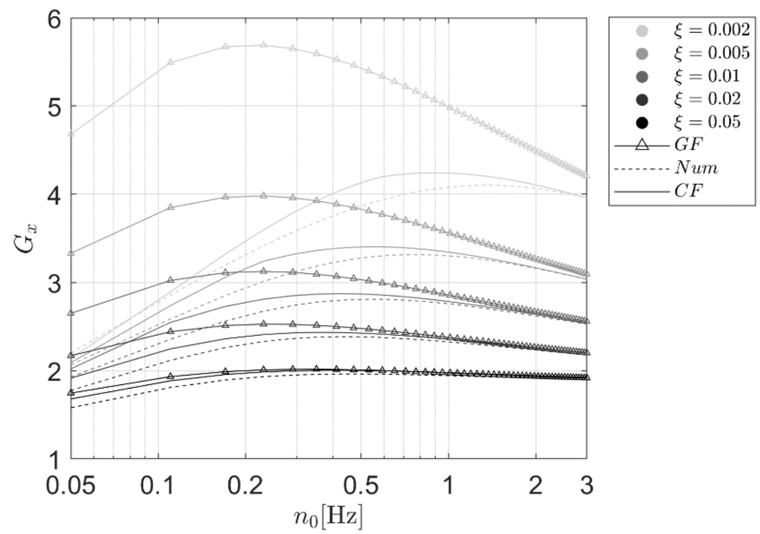

Figure 1. Comparison between the formulation by Kwon and Kareem (2019) (GF, triangles) and the one proposed (continuous lines) along with the one numerically estimated (dashed lines).

5. CONCLUSIONS AND PROSPECTS

The proposed formulation furnishes a more accurate and less conservative estimation of gust response factor with respect to a formulation available in the literature and it is suitable to be adopted for rapid engineering calculations. Future studies aim to generalize the proposed closed-form solution to Multi-Degree-Of-Freedom systems and assess its reliability through data collected from monitored full-scale structures and wind tunnel tests of structural models performed in suitable facilities (e.g. Wind-EEE).

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